U.G. 4th Semester Examination - 2026

MATHEMATICS

[MAJOR-IV]

Course Code : MATH-MJ-T-4

(Differential Equations)

[NEP-2020]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
The notations and symbols have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) Obtain the differential equation of all circles having constant radius 'a'.

b) Examine whether the differential equation
$$ydx + (x + \cos y)\,dy = 0.$$
is exact or not.

c) Find the order and degree of the differential equation
$$\left(1+\left(\frac{dy}{dx}\right)^2\right)^{2/3} = \frac{d^2y}{dx^2}.$$

d) Find the equation of the curve passing through (1, 0) and satisfying the differential equation
$$(1+y^2)\,dx = xy\,dy.$$

e) Solve : $xdy - y\,dx = \cos \frac{y}{x}\,dx$.

f) Substituting $z=\frac{1}{y^2}$, reduce the following differential equation to linear equation and hence find its integrating factor.
$$\frac{dy}{dx} - xy = -e^{-x^2}y^3.$$

[Turn over]

g) Find the partial differential equation by eliminating the arbitrary function F from the equation

$$z = F(x^2 + y^2).$$

h) Solve by Lagrange's method:

$$yzp + zxq = xy, \quad p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}.$$

i) Verify whether z = 0 is an ordinary point or a regular singular point of the differential equation

$$\left(1-z^2\right)\frac{d^2\omega}{dz^2} - z\frac{d\omega}{dz} + 4\omega = 0.$$

j) State Picard's existence and uniqueness theorem.

k) Find the orthogonal trajectories of the family of straight lines $y = mx$.

l) Show that $y^2 = 4ax$ is the singular solution of the differential equation

$$y = px + \frac{a}{p}, \quad p = \frac{dy}{dx}.$$

m) Find the Wronskian of the set of functions

$$\left\{x, x^2, x^3\right\}.$$

n) Given $Q_1(x) = \cos x$ and $Q_2(x) = \sin x$ where $-\infty < x < \infty$. Verify whether they are linearly dependent or linearly independent.

o) Show that $f(x, y) = x^2 + y^2$ satisfies the Lipschitz condition in the rectangle $|x| \le a, |y| \le b$.

2. Answer any **four** questions: 5×4=20

a) Obtain the complete primitive and singular solution of the differential equation

$$y = px + \sin^{-1} p, \quad p = \frac{dy}{dx}.$$

b) Solve by the method of variation of parameters.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x^2.$$

c) Find a complete integral of $px + qy = pq$, by Charpit's method.

d) Prove that e^{x^2} is an integrating factor of $\left(x^2 + xy^4\right)dx + 2y^3dy = 0$ and hence solve it.

e) Find the general solution of the system of equations

$$\frac{dx}{dt} = 7x - y, \quad \frac{dy}{dt} = 2x + 5y.$$

f) Obtain a series solution of the equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

near $x = 0$.

3. Answer any **two** questions: 10×2=20

a) i) Reduce the following equation to Clairaut's form and obtain the complete primitives.

$$xy\,(y - px) = x + py.$$

ii) Solve $x(y^2 + z)\,p - y\,(x^2 + z)q = z\,(x^2 - y^2)$ by Lagrange's method. 5+5

b) i) Solve by the method of undetermined coefficients: $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$.

ii) Apply Charpit's method to find complete integral of $p^2x+q^2y=z$. 5+5

c) i) Show that the function $f(x,y)=(y+y^2)\frac{\cos x}{x^2}$ satisfies Lipschitz condition in $|y|\le 1$ and $|x-1|<\frac{1}{2}$. Also find the Lipschitz constant.

ii) Show that the equation

$$(2x^2+3x)\frac{d^2y}{dx^2}+(6x+3)\frac{dy}{dx}+2y=(x+1)e^x$$

is exact and then solve it. 5+5

d) i) If y_1 and y_2 be two linearly independent solution of the equation $\frac{d^2y}{dx^2}+p\frac{dy}{dx}+Qy=0$, where P and Q are continuous functions of x, then show that $y_1\frac{dy_2}{dx}-y_2\frac{dy_1}{dx}=ce^{-\int pdx}$, where c is a non-zero constant.

ii) By the use of symbolic operator D, find the solution of the equation

$$\frac{d^3y}{dx^3}-5\frac{d^2y}{dx^2}+8\frac{dy}{dx}-4y=e^x+e^{2x}+3e^{-x}.$$

5+5
