U.G. 2nd Semester Examination - 2024

MATHEMATICS

[Skill Enhancement Course (SEC)]

Course Code : MATH-SEC-T-02

(Fuzzy Set Theory)

[NEP-2020]

Full Marks : 35 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Sysmbols and notations have their usual meanings.

1. Answer any **five** questions : $1\times5=5$

a) Find $B-A$, where the interval numbers are given by $A=[-6,5]$ and $B=[-2,8]$.

b) Find ${}^{0.6+}A$ of the fuzzy set $A=\frac{0.9}{x_1}+\frac{0.7}{x_2}+\frac{0.7}{x_3}$.

c) Using standard fuzzy union, find $A\cup B$, where

$A=\frac{0.7}{0}+\frac{0.4}{3}+\frac{0.9}{4}+\frac{0.1}{6}$ and

$B=\frac{0.6}{0}+\frac{0.1}{3}+\frac{0.7}{4}+\frac{0.6}{6}$.

d) Find the height of the fuzzy set $A\cup B$ which is found from Q. 1(c).

e) From membership matrix of a binary fuzzy relation, how can you identify a fuzzy compatibility or fuzzy tolerance relation?

[Turn over]

f) Define fuzzy number.

g) Give an example to show that the standard intersection of two normal fuzzy sets may not be normal.

h) Find max-min composition $Q \cdot P$ of the binary relations $Q(X,X) = (0.2 \quad 0.4)$ and

$$P(X,X) = \begin{pmatrix} 0.5 & 0.9 \\ 0 & 0.3 \end{pmatrix}.$$

2. Answer any **two** questions : $5 \times 2 = 10$

a) Let A and B be fuzzy sets defined on the universal set $\mathbb{Z}$ whose membership functions are given by $A(x) = \frac{0.3}{-3} + \frac{0.4}{0} + \frac{0.2}{2} + \frac{0.9}{5}$ and $B(x) = \frac{0.2}{-5} + \frac{1}{3} + \frac{0.9}{0} + \frac{0.7}{8}$. Also, let a function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined for all $x_1, x_2 \in \mathbb{Z}$ by $f(x_1, x_2) = x_1 \cdot x_2$. Calculate $f(A,B)$.

b) Find the transitive max-min closure of the fuzzy relation: $R = \begin{bmatrix} 0.7 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.8 & 1 \\ 0 & 0.4 & 0 & 0.5 \\ 0 & 0.4 & 0.8 & 0 \end{bmatrix}$.

c) Solve the fuzzy equation: $A \cdot X = B$ where A, B are given by

$$A(x) = \begin{cases} \frac{x-3}{2} & if\ 3 \le x \le 5 \\ \frac{8-x}{3} & if\ 5 \le x \le 8 \\ 0 & otherwise \end{cases} \text{ and } B(x) = \begin{cases} \frac{x-12}{8} & \text{if } 12 \le x \le 20 \\ \frac{32-x}{12} & \text{if } 20 \le x \le 32 \\ 0 & \text{otherwise} \end{cases}.$$

3. Answer any **two** questions : $10 \times 2 = 20$

a) i) State and prove second decompostion theorem.

ii) Find the relational join $P * Q$ of the following fuzzy relation and hence find composition $P \circ Q$:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0.5 & 0.9 & 0.2 \\ 0.6 & 0 & 0.8 \\ 0.2 & 0 & 0.1 \\ 0.4 & 0.3 & 0 \end{bmatrix} \end{array} \text{ and } Q = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cc} \alpha & \beta \end{array} \\ \begin{bmatrix} 0.3 & 0.5 \\ 0 & 0.2 \\ 0.9 & 1 \end{bmatrix} \end{array}.$$

5+5

b) i) Find $B - A$ from the following two fuzzy numbers:

$$A(x) = \begin{cases} \frac{3x+5}{6}, & -\frac{5}{3} \le x < \frac{1}{3} \\ \frac{4-3x}{3}, & \frac{1}{3} \le x < \frac{4}{3} \\ 0, & \text{Otherwise} \end{cases} \text{ and } B(x) = \begin{cases} \frac{x-4}{3}, & 4 \le x < 7 \\ \frac{11-x}{4}, & 7 \le x < 11 \\ 0, & \text{Otherwise} \end{cases}$$

ii) Show by an example that if a fuzzy binary relation R is reflexive, then $R \subset R \circ R$.

5+5

c) i) Establish the relationship between $\cap_{i\in I}{}^{\alpha+} A_i$ and ${}^{\alpha+}[\cap_{i\in I} A_i]$., where $A_i \in \mathcal{F}(X)$ for all $i \in I$, where I is an index set.

ii) Consider the fuzzy set $A(x) = \frac{x}{x+2}$ defined on the universal $X = \{0, 1, 2, ..., 10\}$, and let $f(x) = x^2$ for all $x \in X$. Use extension principle to derive $f(A)$. 5+5
