# U.G. 2nd Semester Examination - 2024

## MATHEMATICS

### [MINOR]

**Course Code : MATH-MI-T-1**

**(Algebra and Analytical Geometry)**

**[NEP-2020]**

Full Marks : 40 Time : 2 Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*The notations and symbols have their usual meanings.*

1. Answer any **five** questions : $2\times5=10$

a) Find the cube roots of 1.

b) Find the remainder when $4x^5+3x^3+6x^2+5$ is divided by $2x+1$.

c) Prove that an invertible matrix unique inverse.

d) A relation $\rho$ is defined on the set $\mathbb{N}$ by "$m\rho n$" if and only if $m$ is divisor of $n$" for $m, n \in \mathbb{N}$. Prove that $\rho$ is transitive but not symmetric.

e) To what point the origin is to be moved so that one can get rid of the first degree terms from the equation $x^2+xy+2y^2-7x-5y+12=0$?

[Turn over]

f) Does the equation $x^2+3xy+2y^2=0$ represents a pair of straight line? If so what is the angle between them?

g) Find the equation of the parabola whose focus is the origin and whose directrix is the straight line $2x+y=1$.

h) Find the points on the conic $\frac{5}{r}=1+2\cos\theta$, whose radius vector is 5.

2. Answer any **two** questions : $5\times2=10$

a) If $\cos\theta=\frac{1}{2}(a+\frac{1}{a})$, $\cos\phi=\frac{1}{2}(b+\frac{1}{b})$, show that one of the values of $\cos(\theta+\phi)$ is

$$\frac{1}{2}(ab+\frac{1}{ab}).$$

b) If the equation $x^4+ax^3+bx^2+cx+d=0$ has three equal roots, prove that each of them is equal to $\frac{6c-ab}{3a^2-8b}$.

c) If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angles between the other pair, then prove that $pq=-1$.

d) Show that the pole of any tangent to the hyperbola $xy=c^2$ with respect to the circle $x^2+y^2=a^2$ lies on a concentric and similar hyperbola.

3. Answer any **two** questions : $10\times2=20$

a) i) Define rank of a matrix. Find the rank of the matrix

$$\begin{pmatrix} 2 & 4 & 3 \\ 4 & 6 & 3 \\ 3 & 3 & 1 \end{pmatrix}.$$ 1+4

ii) Solve the following system of equations

$$x+y-z=6$$
$$2x-3y+z=-1$$
$$3x-4y+2z=-1.$$ 5

b) i) For what integral values of $m$, $x^2+x+1$ is a factor of $x^{2m}+x^m+1$? 5

ii) Define an abelian group. Show that the set $G$ of all numbers of the form $2^n$, where $n$ is an integer, is an abelian group with respect to ordinary multiplication. 1+4

c) i) Reduce the following equation in canonical form and determine the nature of the conic

$$x^2-2xy+2y^2-4x-6y+3=0.$$ 5

ii) Prove that the equation of the chord of the conic $\frac{l}{r}=1+e\cos\theta$ joining the points whose vectorial angles are $\alpha-\beta$ and $\alpha+\beta$ is $\frac{l}{r}=e\cos\theta+\sec\beta\cos(\theta-\alpha)$. 5

d) i) If the tangents at $P$ and $Q$ of a parabola meet at a point $T$, and $S$ be the focus of the parabola, then show that $ST^2 = SP.SQ$. 5

ii) If the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

represents two straight lines, then prove that the product of the lengths of the perpendiculars from the origin to these straight line is

$$\frac{c}{\sqrt{(a-b)^2 + 4h^2}}$$ 5

---