U.G. 1st Semester Examination - 2025

MATHEMATICS

[Skill Enhancement Course (SEC)]

Course Code : MATH-SEC-T-01

(Logic & Boolean Algebra)

[NEP-2020]

Full Marks : 35 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Symbols / Notations have their usual meanings.

1. Answer any **five** questions: $1\times5=5$

   a) If $P(x, y)$ is "$x^2 + y = 12$", then find the truth value of $P(2,8)$ and $P(1,6)$.

   b) Define negative proposition $(\neg p)$ with its truth table.

   c) Prove that $\neg\neg p \Leftrightarrow p$.

   d) Give an example of a distributive lattice.

   e) If $(L_1, \leq_1, \vee_1, \wedge_1)$ and $(L_2, \leq_2, \vee_2, \wedge_2)$ are two lattices, how do you define *meet* $(\wedge)$ and *join* $(\vee)$ for the product lattice $L = L_1 \times L_2$?

   f) In Boolean algebra $(B, +, 0, 1, ')$, show that $1' = 0$ and $0' = 1$.

[Turn over]

g) Is the Boolean function $f(x, y, z) = (x \cdot y' + y \cdot z')' + x \cdot z$ in Disjunctive Normal Form (*D.N.F.*)?

h) Differentiate between *OR* gate and *AND* gate.

2. Answer any **two** questions: 5×2=10

a) Using Precedence rules of logical operators, insert the parentheses in the compound expression $p \to q \vee r \wedge \neg p \leftrightarrow \neg r$. Write its truth table. 2+3

b) Define binary compositions in the power set $P(X)$ of a nonempty set X to make $P(X)$ a Boolean algebra with justification. 5

c) In Boolean algebra $(B,+,0,1,')$, prove the associative law:

$$a+(b+c)=(a+b)+c$$

and then using duality principle of Boolean algebra, establish

$$a \cdot (b \cdot c)=(a \cdot b) \cdot c.$$ 4+1

d) In a lattice $(L,\leq,\vee,\wedge)$, prove that $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$. 5

3. Answer any **two** questions: 10×2=20

a) i) Consider the argument:

"If I am guilty, then I must be punished: I must not be punished. Thus, I am guilty." Express the argument in a compound proposition. Is the argument logically correct?

ii) Simplify the following Boolean expression and then draw a circuit for it:

$(x \cdot y + z') \cdot (y' + x' \cdot z) + x'$. (2+3)+5

b) i) What is the *"duality principle"* of propositions. Find dual of

$(p \wedge q) \vee p$

and using duality principle, show that

$(p \wedge q) \vee p \equiv p$.

ii) Without using truth table, find the *Full Disjunctive Normal Form* (*F.D.N.F.*) of the Boolean function

$f(x,y,z) = x + (x' \cdot y' + x' \cdot z)'$. (4+1)+5

c) i) In a distributive lattice $(L,\leq,\vee,\wedge)$, prove that $a \vee (a \wedge b) = a$

and then

$a \vee b = a \vee c,\ a \wedge b = a \wedge c \Rightarrow b = c$.

ii) Convert the *Full Disjunctive Normal Form* (*F.D.N.F.*)

$f(x,y,z) = x \cdot y' \cdot z' + x' \cdot y \cdot z' + x' \cdot y' \cdot z + x \cdot y \cdot z$ to *Full Conjunctive Normal Form* (*F.C.N.F*). (3+3)+4

d) i) In a lattice $(L, \leq, \vee, \wedge)$, prove that $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b$.

ii) Show that the compound proposition $(p \wedge q) \vee p \leftrightarrow \neg p$ is a contradiction.

5+5