# U.G. 1st Semester Examination - 2025

# MATHEMATICS

## [MINOR]

## Course Code : MATH-MI-T-1

## (Algebra and Analytical Geometry)

## [NEP-2020]

Full Marks : 40 Time : 2 Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*The notations and symbols have their usual meanings.*

1. Answer any **five** questions: $2\times5=10$

   a) Give the polar representation of the complex number: $z=-1-i$.

   b) Show that the equation $x^2+3xy+2y^2=0$ represents a pair of straight lines.

   c) Show that $G=\{2^n : n=0,\pm1,\pm2,\pm3,...\}$ is a multiplicative group.

   d) Find the transformed form of the equation $\frac{x}{a}+\frac{y}{b}=2$ when the origin is transferred to $(a, b)$.

   e) Find the equation of the sphere with centre at $(0, 0, 1)$ and radius 1.

[Turn over]

f) Define a relation $\rho$ on $Z$ by $a\rho b$ if and only if $3a+4b=7n$, for some integer $n$. Check whether $\rho$ is transitive.

g) Find the condition so that the rank of $\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}$ is 3.

h) Find the value of $\alpha\beta+\alpha\gamma+\beta\gamma$ if $\alpha,\beta,\gamma$ are the roots of $x^3-6x^2+11x-6=0$.

2. Answer any **two** questions: $5\times2=10$

a) Find for what values of $a$, $b$ the following system
$x + 4y + 2z = 1$,
$2x + 7y + 5z = 2b$,
$4x + ay + 10z = 2b - 1$, has a unique solution. 5

b) Expand $Cos^7\theta$ in a series of cosines of multiples of $\theta$. 5

c) Show that the equation to the pair of straight lines through the origin perpendicular to the pair of straight lines $ax^2 + 2hxy + by^2 = 0$, is $bx^2 - 2hxy + ay^2 = 0$. 5

d) If by a rotation of rectangular axes about the origin, the expression $(ax^2 + 2hxy + by^2)$ changes to $(AX^2 + 2HXY + BY^2)$, then prove that $a + b = A + B$ and $ab - h^2 = AB - H^2$. 5



3. Answer any **two** questions: $10\times2=20$

a) i) If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$, then prove that $\cos(\beta+\gamma)+\cos(\gamma+\alpha)+\cos(\alpha+\beta)=0$. 5

ii) Apply Descartes' rule of signs to examine the nature of the roots of the equation $x^4+2x^2+3x-1=0$. 5

b) i) Show that the map $f(x)=\dfrac{1}{1+e^{-x}}$ is a bijection from $R$ to $(0,1)$. 4

ii) What do you mean by a subgroup of a group $G$? Do you find a group structure in $Z$? If so then check whether $H = \{x + 2y: x, y \in Z\}$ is a subgroup of $Z$ or not. If $A$, $B$ are to subgroups of $Z$, then show that $A\cap B$ is also a subgroup of $Z$. Here $Z$= set of all integers. 6

c) i) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\dfrac{l}{r}=1+e\cos\theta$, if $(l\cos\alpha-ep^2)^2+l^2\sin^2\alpha=p^2$. 6

ii) Find the values of $b$ and $g$ such that the equation $4x^2 + 8xy + by^2 + 2gx + 4y + 1 = 0$ represents a conic having infinitely many centres. 4

d) i) Find the equation of the sphere touching the three coordinate planes. 5

ii) Find the equation of the right circular cone which contains the three positive coordinate axes. 5

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