U.G. 1st Semester Examination - 2025

PHYSICS

[MINOR]

Course Code : PHY-MI-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: 1×5=5

a) State Gauss's divergence theorem.

b) Write down the order and degree of the differential equation $y = x\frac{dy}{dx} + 3\frac{dx}{dy}$.

c) Check the continuity of the function $f(x) = |x| - x$ at $x = 0$.

d) Write down the Taylor series expansion of $f(x)$ around $x = a$.

e) Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

[Turn over]

f) If $f(x,y,z) = 3x^2y - y^3z^2$, find $\vec{\nabla} f$ at the point $(1, -2, -1)$.

g) Find the trace and eigenvalues of the matrix $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$.

h) Define Dirac delta function.

GROUP-B

2. Answer any **three** questions: 5×3=15

a) Solve the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2e^x$, subject to the boundary condition $x = 0, y = 1$ and $\frac{dy}{dx} = 1$. 5

b) Verify Green's theorem in the plane for $\oint_c (xy + y^2)\, dx + x^2\, dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5

c) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix}$. 5

d) Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$. 5

e) i) Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

ii) If $\vec{A} = x^2y\,\hat{\imath} - 2xz\,\hat{\jmath} + 2yz\,\hat{k}$, find curl $\vec{A}$. 3+2

f) Prove the vector identity $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$. 5

GROUP-C

3. Answer any **one** question: 10×1=10

a) i) Find the velocity and acceleration of a particle which moves along the curve $x = 2sin3t,\ y = 2cos3t,\ z = 8t$ at any time $t > 0$. Find also the magnitude of the velocity and acceleration.

ii) Find the volume of the parallelopiped whose edges are given by $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, $\vec{B} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{C} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

iii) Represent delta function as a limit of a Gaussian function and rectangular function.

iv) Prove $\delta(x^2 - a^2) = \frac{\delta(x-a)+\delta(x+a)}{2|a|}$.

3+2+2+3

b) i) Evaluate $\iint_S \vec{F}.\hat{n}\, dS$, where $\vec{F} = 4xz\,\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ and S is the surface of the cube bounded by $x=0, x=1,\ y=0, y=1, z=0, z=1$.

ii) Find the particular integral of the differential equation $(D^2+1)y = cosx$.

iii) Find the primitive of the differential equation $xy - \frac{dy}{dx} = y^3e^{-x^2}$. 3+3+4
