U.G. 1st Semester Examination - 2025

PHYSICS

[MAJOR]

Course Code : PHY-MJ-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions : $2\times5=10$

   a) If A= 2i − 3j + k and A . B = 0, does it follow that B = 0?

   b) Find the slope of the line whose parametric equation is $\mathbf{r} = (\mathbf{i}-\mathbf{j})+(2\mathbf{i}+3\mathbf{j})t$.

   c) Show that the functions $\cos x$ and $\sin x$ are linearly independent.

   d) Show that if A and B are Hermitian, then AB is not Hermitian unless A and B commute.

   e) If $s = t^u$, find $\partial s/\partial t, \partial s/\partial u$.

f) Find the gradient of $w = x^2y^3z$ at $(1, 2, -1)$.

g) Evaluate $\nabla \times \left(\frac{\mathbf{r}}{|\mathbf{r}|}\right)$

h) Show that $x\delta'(x) = -\delta(x)$.

i) Three coins are tossed; what is the probability that two are heads and one tails?

GROUP-B

2. Answer any **two** questions: 5×2=10

a) i) The rate at which a radioactive substance decays is proportional to the remaining number of atoms. If there are N_0 atoms at $t = 0$, find the number at time t.

ii) Solve $\frac{dy}{dx}$ - xy = x, y = 1 when x = 0.

2+3

b) $\oiint \mathbf{r}\cdot\mathbf{n}\,ds$ is over the whole surface of the cylinder bounded by $x^2+y^2 = 1, z = 0$ and $z = 3$.

c) If $u = f(x - ct) + g(x + ct)$, show that $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$.

d) Solve the differential equation $(D^2 + 5D + 4)y = \cos 2x$.

e) Evaluate :

i) $\int_0^{\pi} \sin x\, \delta(x - \frac{\pi}{2})\, dx$

ii) $\int_{-1}^{1} e^{3x}\delta'(x)\, dx$

GROUP-C

Answer any **two** questions : 10×2=20

3. a) $\oiint_A (2x - 3y)\, dx\, dy$, where A is the triangle with vertices (0, 0), (2, 1), (2, 0).

b) Find the Jacobian of x, y with respect to the polar coordinates r, θ. 6+4

4. a) Find the gradient of $\phi = z\sin y - xz$ at the point $(2, \pi/2, -1)$. Starting at this point, in what direction is ϕ *decreasing* most rapidly? Find the derivative of ϕ in the direction $2\mathbf{i} + 3\mathbf{j}$.

b) Evaluate the line integral $\int_C y^2 dx + 2x\, dy + dz$, where C connects (0, 0, 0) with (1,1,1) along straight lines from (0, 0, 0) to (1, 0, 0) to (1, 0, 1) to (1, 1, 1). 4+6

5. a) Find the eigenvalues and eigenvectors of the following matrix.

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

b) Find the eigenvalues and eigenvectors of the real symmetric matrix

$$\mathrm{M} = \begin{pmatrix} A & H \\ H & B \end{pmatrix}$$

Show that the eigenvalues are real and the eigenvectors are perpendicular. 6+4

6. a) For a simple closed curve C in the plane show by Green's theorem that the area inclosed is

$$A = \frac{1}{2}\oint_C (x\,dy - y\,dx).$$

b) Show that the area inside the ellipse $x = a\cos\theta$, $y = b\sin\theta$, $0 \le \theta \le 2\pi$, is $A = \pi ab$. 5+5