U.G. 6th Semester Examination - 2026
MATHEMATICS
[MAJOR-X]
Course Code : MATH-M-T-10
(Multivariate Calculus, Vector and Tensor Analysis)
[NEP-2020]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
The notations and symbols have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

a) Determine whether $\lim\limits_{\substack{x\to0\\y\to0}}\frac{xy}{x^2+y^2}$ exists or not with justification.

b) Prove that the function

$$f(x,y)=\frac{xy}{\sqrt{x^2+y^2}},\ (x,y)\neq(0,0)$$
$$=0\ (x,y)=(0,0)$$

is continuous at (0, 0).

c) Determine the total differential of the function $f(x,y)=\sqrt{|xy|}$ at (0, 0).

d) Show that the functions $u=x+y-z,\ v=x-y+z,\ w=x^2+y^2+z^2-2yz$ are not independent.

e) Define Kronecker symbol $\delta^i_j, i, j=1, 2, \ldots n$. show that $\delta^i_i=n$.

f) State necessary conditions for $f(a, b)$ to be an extreme value of a function $f(x, y)$.

[Turn over]

g) Prove that the area bounded by the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is πab square units.

h) Find the length of the arc of the catenary $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ from the vertex to the point (x_1, y_1).

i) Find the volume of a paraboloid formed by revolving the parabola $y^2 = 4ax$ about the *x-axis* and bounded by the section $x = x_1$.

j) In what direction, from the point (1, 1, –1), is the directional derivative of $f = x^2 - 2y^2 + 4z^2$ maximum? Also find the value of this maximum directional derivative.

k) A particle moves along the curve $x = 4\cos t$, $y = 4\sin t$, $z = 6t$. Find the magnitudes of the velocity and acceleration at any time t.

l) If the relation $b^i_j v_i = 0$ holds for an arbitrary covariant vector v_i, show that $b^i_j = 0$.

m) If a tensor T_{ijk} is symmetric in the first two indices from the left and skew-symmetric in the second and third indices from the left, show that $T_{ijk}=0$.

n) If A_i and B_j are two covariant vectors, show that $g^{ij}(A_iB_j - A_jB_i) = 0$, where g^{ij} is contravariant metric tensor.

o) Show that the components of a tensor of type (0, 2) can be expressed as the sum of a symmetric tensor and a skew-symmetric tensor of same type.

2. Answer any **four** questions: 5×4=20

a) Define a homogeneous function of degree n with an example. If $u=f(x, y)$ be a homogeneous function of two independent variables x, y of degree n, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=n.u$.

b) Consider the equation $\frac{x}{a+k}+\frac{y}{b+k}+\frac{z}{c+k}=1$ with K as the variable. If λ, μ, ν are the roots (assumed to be all real) of the above equation, prove that $\frac{\partial(x,y,z)}{\partial(\lambda,\mu,\nu)}=\frac{-(\mu-\nu)(\nu-\lambda)(\lambda-\mu)}{(b-c)(c-a)(a-b)}$

c) Evaluate $\iint\limits_R \frac{\sqrt{a^2b^2-b^2x^2-a^2y^2}}{a^2b^2+b^2x^2+a^2y^2}\,dxdy$, the region of integration being R, the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

d) Evaluate $\int\limits_\Gamma (\cos x\sin y - xy)dx+\sin x\cos y\,dy)$, where Γ is the circle $x^2+y^2=1$ in the xy-plane described in the positive sense, using Green's theorem.

e) If a_{ij} and b_{ij} are two symmetric tensors and u^i, v^i are components of two contravariant vectors satisfying $(a_{ij} - kb_{ij})\, v^i = 0$, $(a_{ij} - k'\, b_{ij})\, v^i = 0$, i, $j = 1, 2, \ldots, n$, $k \neq k^1$. Show that $b_{ij}u^iv^j = 0$ and $a_{ij}u^iv^j = 0$.

f) If B_{ij} are components of a covariant tensor of second order and c^i, d^j are components of two contravariant vectors, show that $B_{ij}C^iD^j$ is an invariant.

3. Answer any **two** questions: 10×2=20

a) i) A differentiable function $f(x, y)$, when expressed in terms of the new variables u and v defined by $x=\frac{1}{2}(u+v), y=\sqrt{uv}$

becomes g(*u*, *v*). Prove that

$$\frac{\partial^2 y}{\partial u \partial v} = \frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + 2\frac{x}{y}\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} + \frac{1}{y}\frac{\partial f}{\partial y}\right).$$

ii) Find the point where the function $x^2+y^2+z^2$ has a minimum subject to $ax+by+cz = p$.

7+3

b) i) Evaluate $\iint_S \left(x^3 dydz + y^3 dzdx + z^3 dxdy\right)$ over the sphere S : $x^2 + y^2 + z^2 = a^2$.

ii) Find the volume of the solid generated by revolving the cardioide r = a(1− cosθ) about the initial line.

5+5

c) i) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C (xdy-ydx)$.

Hence find the area of the region bounded by the curve $x = \cos\theta$, $y = \sin\theta$.

ii) Apply Stoke's theorem to prove that

$$\int_C (ydx + zdy + xdz) = -2\sqrt{2}\pi a^2,$$

where C is the curve given by $x^2 + y^2+z^2 - 2ax - 2ay = 0, x + y = 2a$ and begins at the point (2a, 0, 0) and goes at first below the *z*-plane.

4+6

d) i) If, g_{ij} denotes metric tensor of type (0, 2), then show that $[jk, i] - [ij, k] = \frac{\partial g_{ii}}{\partial x^k} - \frac{\partial g_{ik}}{\partial x^i}$

ii) Define covariant derivative of a tensor of type (0, 2). Prove that covariant derivative of the metric tensor g_{ij} is zero.

4+(2+4)