

U.G. 6th Semester Examination - 2026**MATHEMATICS****[MAJOR-VIII]****Course Code : MATH-M-T-08****(Mechanics-I)****[NEP-2020]**

Full Marks : 60

Time : $2\frac{1}{2}$ Hours*The figures in the right-hand margin indicate marks.**The notations and symbols have their usual meanings.*

1. Answer any ten questions: $2 \times 10 = 20$
- a) The law of motion in a straight line being $s = vt^2$, show that the acceleration is constant.
 - b) Prove that at an apse, the planet moves at right angle to the radius vector.
 - c) What is an impulsive force? How is it measured?
 - d) The velocity of a particle moving in a straight line is given by $\dot{s} = ks^2 + k$, prove that its acceleration varies as the distance from a fixed point in the line.
 - e) When is the equilibrium of a heavy body resting on a fixed body said to be unstable?

[Turn over]

- f) Find the moment of inertia of the perimeter of a circle about a tangent.
- g) Obtain the relation between the rate of change of angular momentum of a moving particle and the force acting on it.
- h) A particle describes a curve $r = ae^{\theta}$ with constant angular velocity. Show that its transverse acceleration varies as the distance from the pole.
- i) Define angular momentum of a particle.
- j) Explain the terms "apse" and "apsidal angle" related to a central orbit.
- k) Obtain the relation between the rate of change of angular momentum of a moving particle and the force acting on it.
- l) What is meant by central force? Give one example.
- m) What is a compound pendulum?
- n) State the conditions of equilibrium of a rigid body under coplanar forces.
- o) Define impulse of a force.
2. Answer any **four** questions: 5×4=20
- a) Discuss the contexts where the terms *terminal velocity* and *escape velocity*, appear respectively.

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- b) A particle describes a central orbit under an attractive force F per unit mass. Establish the differential equation of its path in the form $\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}$. 5
- c) If V_1 and V_2 are the velocities of a planet when nearest and farthest from the sun, prove $(1 - e)V_1 = (1 + e)V_2$ where e is the eccentricity 5
- d) Write down the expressions of potential energy and kinetic energy of a simple pendulum of length l oscillating in a uniform gravitational field. 5
- e) Prove that a particle moving under a central force describes equal areas in equal times. 5
- f) Obtain the time period of a simple pendulum. 5
- g) Show that for inverse square law of attraction the orbit is a conic section. 5
3. Answer any **two** questions: 10×2=20
- a) A particle moves in a plane such that its tangential and normal accelerations are always equal and the velocity varies as the arc length from a fixed point. Obtain the path. 10

b) Find the moment of inertia of a uniform rod about an axis through one end perpendicular to its length. Hence obtain the kinetic energy of rotation. 10

c) A body of mass $(m_1 + m_2)$ is split into two parts by an internal explosion generating kinetic energy E . If the parts move in the same line as before, show that their relative velocity is

$$\sqrt{\frac{2E(m_1 + m_2)}{m_1 m_2}}. \quad 10$$

d) Obtain tangential and normal components of velocity and acceleration of a particle moving along a plane curve. 10