U.G. 2nd Semester Examination - 2025

MATHEMATICS

[Skill Enhancement Course (SEC)]

Course Code : MATH-SEC-T-02

(Fuzzy Set Theory)

[NEP-2020]

Full Marks : 35 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Sysmbols and notations have their usual meanings.

1. Answer any **five** questions : $1\times5=5$

a) Find the sum of the two interval numbers, A= [–5, 10] and B = [2, 8].

b) Find $^{0.5}A$ of the fuzzy set $A = \frac{0.7}{x_1} + \frac{0.2}{x_2} + \frac{0.5}{x_3} + \frac{0.55}{x_4}$.

c) Using standard fuzzy intersection, find $A \cap B$ defined on $\mathbb{R}$, where $A = \frac{0.3}{0} + \frac{0.8}{1} + \frac{0.9}{7} + \frac{0.5}{2}$ and $B = \frac{0.6}{1} + \frac{0.1}{3} + \frac{0.6}{2} + \frac{0.6}{7}$.

d) State 1st decomposition theorem of fuzzy set theory.

e) How can you check a fuzzy relation reflexive from a membership matrix?

f) Find max-min composition $Q.P$ of the binary relations $Q(X, X)=(0.3\ 0.2)$ and

$$P(X,X) = \begin{pmatrix} 0.6 & 0.4 \\ 0 & 0.3 \end{pmatrix}.$$

g) Find the level set of the fuzzy number $A(x) = \frac{x}{x+4}$ defined on $\{0, 1, 2, 3,\}$.

h) Give an example which is a fuzzy set but not a fuzzy number.

2. Answer any **two** questions : 5×2=10

a) Find strong α–cut, height and support of the following fuzzy set

$$A(x) = \begin{cases} 0 & if\ x \leq 5\ or\ x \geq 50 \\ \dfrac{x-5}{15} & if\ 5 \leq x \leq 20 \\ \dfrac{50-x}{30} & if\ 20 \leq x \leq 50 \end{cases}.$$

b) Find the standard max-min composition $P \circ Q$ of the following two binary fuzzy relations P and Q:

$$P = \begin{matrix} & a & b & c \\ 1 & 0.3 & 0.5 & 0 \\ 2 & 0.3 & 0 & 0 \\ 3 & 0 & 0.8 & 0.7 \\ 4 & 0.8 & 0 & 0.4 \end{matrix} \quad \text{and } Q = \begin{matrix} & \alpha & \beta \\ a & 0.3 & 0.6 \\ b & 0.7 & 0 \\ c & 0.5 & 0.9 \end{matrix}.$$

c) Establish the relationship between $\bigcup_{i\in I} {}^{\alpha}A_i$ and ${}^{\alpha}(\bigcup_{i\in I} A_i)$, where $A_i \in \mathcal{F}(X)$ for all $i \in I$, where I is an index set.

3. Answer any **two** questions : 10×2=20

a) i) What is extension principle of fuzzy set? Find the relationship between ${}^{\alpha}f(A)$ and $f({}^{\alpha}A)$. Justify whether those are equal.

ii) Find the standard max-min composition $P \circ Q$ of the following two binary fuzzy relations P and Q.

$$P = \begin{matrix} & a & b & c \\ 1 & 0.3 & 0.5 & 1 \\ 2 & 0.3 & 0 & 0 \\ 3 & 0 & 0.8 & 0.7 \\ 4 & 0.8 & 0.2 & 0.4 \end{matrix} \quad \text{and } Q = \begin{matrix} & \alpha & \beta \\ a & 0.3 & 0.6 \\ b & 0.7 & 0 \\ c & 0 & 0.9 \end{matrix}.$$

(2+3)+5

b) i) Solve $A + X = B$ where the fuzzy numbers are given by:

$$A(x) = \begin{cases} \frac{x-18}{18}, & 18 \leq x < 36 \\ \frac{50-x}{14}, & 36 \leq x < 50 \\ 0, & \text{otherwise} \end{cases} \quad \text{and } B(x) = \begin{cases} \frac{x-4}{3}, & 4 \leq x < 7 \\ \frac{11-x}{4}, & 7 \leq x < 11 \\ 0, & \text{Otherwise} \end{cases}$$

ii) Using 2nd decomposition process, decompose the fuzzy set

$$F = \frac{0.4}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.8}{d} + \frac{1}{e} + \frac{0.2}{f}$$

5+5

c) i) Check the following binary fuzzy relation and find possible partitions of that relation:

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} \text{a} & \text{b} & \text{c} & \text{d} \end{array} \\ \begin{bmatrix} 1 & 0.2 & 0.2 & 0.8 \\ 0.2 & 1 & 0.6 & 0.2 \\ 0.2 & 0.6 & 1 & 0.2 \\ 0.8 & 0.2 & 0.2 & 1 \end{bmatrix} \end{array}.$$

ii) Find the transitive max-min closure of the following fuzzy relation:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0.8 & 0 \\ 1 & 1 & 0.8 & 0.7 \\ 0.8 & 0.8 & 1 & 0.7 \\ 0 & 0.7 & 0.7 & 1 \end{bmatrix} \end{array}$$

5+5
