

U.G. 2nd Semester Examination - 2025

MATHEMATICS

[MINOR]

Course Code : MATH-MI-T-1

(Algebra and Analytical Geometry)

[NEP-2020]

Full Marks : 40

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.**The notations and symbols have their usual meanings.*

1. Answer any five questions: $2 \times 5 = 10$
- Find the principal value of i^i .
 - If the sum of roots of the equation $x^2 + px^2 + qx + r = 0$ is zero, prove that $pq = r$.
 - Show that the product of two orthogonal matrices of the same order is orthogonal.
 - If A and B are any two sets, prove that $A - B$ and $A \cap B$ are disjoint.
 - Determine the nature of the conic $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$.

[Turn over]

- f) Show that the equation
 $x^2 + 2\sqrt{3}xy + 3y^2 - 3x - 3\sqrt{3}y - 4 = 0$
 represents a pair of parallel straight lines.
- g) Give an example of a relation on a set which is reflexive and symmetric but not transitive.
- h) Obtain the equation of the straight line
 $\frac{x}{a} + \frac{y}{b} = 2$, when the origin is shifted to the point
 (a, b).

2. Answer any two questions: $5 \times 2 = 10$

- a) If α and β are the roots of the equation
 $x^2 - 2x \cos \theta + 1 = 0$, find the equation whose
 roots are α^n and β^n .
- b) If α, β, γ are the roots of the equation
 $x^3 + px + q = 0$, find the value of

i) $\sum \frac{1}{\alpha + \beta}$

ii) $(\beta + \gamma - 2\alpha)(\gamma + \alpha - 2\beta)(\alpha + \beta - 2\gamma)$

- c) Prove that the pair of straight lines joining the origin to the points of intersection of the curves

$$ax^2 + 2hxy + by^2 + 2gx = 0 \text{ and}$$

$$a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$$

are perpendicular if $g'(a+b) = g(a'+b')$.

- d) Prove that the equation of the chord of the conic
 $\frac{l}{r} = 1 + e \cos \theta$ joining the points whose vectorial
 angles are $\alpha - \beta$ and $\alpha + \beta$ is
 $\frac{l}{r} = e \cos \theta + \sec \beta \cos(\theta - \alpha)$.

3. Answer any two questions: $10 \times 2 = 20$

- a) i) Determine the rank of the following matrix
 by considering different values of λ :

$$\begin{bmatrix} \lambda & 1 & 0 \\ 3 & \lambda - 2 & 1 \\ 3(\lambda + 1) & 0 & \lambda + 1 \end{bmatrix}$$

- ii) For what of K will the system

$$2x_1 + x_2 = 5$$

$$x_1 + 3x_2 = -1$$

$$3x_1 + 4x_2 = K$$

be solvable? For that values of K solve the system. $5+5$

- b) i) On the set of integers \mathbb{Z} , the binary operation * defined by $a * b = a + b - 2$ for all $a, b \in \mathbb{Z}$. Show that $(\mathbb{Z}, *)$ is a group.

ii) Prove that the roots of the equation

$$\frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} = x \text{ are all real. } 5+5$$

c) i) If the pair of the straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq = -1$.

ii) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to canonical form and determine the nature of the conic. 5+5

d) i) Find the area of the portion of the circle $x^2 + y^2 = 1$, which lies inside the parabola $y^2 = 1 - x$.

ii) Show that the locus of the point of intersection of two tangents to $\frac{l}{r} = 1 + e \cos \theta$, which are at right angles to one another is

$$r^2(e^2 - 1) - 2ler \cos \theta + 2l^2 = 0. \quad 5+5$$
