U.G. 6th Semester Examination - 2025

MATHEMATICS

[PROGRAMME]

Discipline Specific Elective (DSE)

Course Code : MATH-G-DSE-T-02(A)&(B)

[Old Syllabus]

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
Symbols and Notations have their usual meanings.

Answer all the questions from Selected Option.

OPTION–A

MATH-G-DSE-T-02A

(Linear Programming)

1. Answer any **ten** questions : $2\times10=20$

   a) State fundamental theorem of L.P.P.

   b) State the fundamental theorem of duality.

   c) Prove that dual of the dual is the primal.

   d) Write the standard form of an assignment problem.

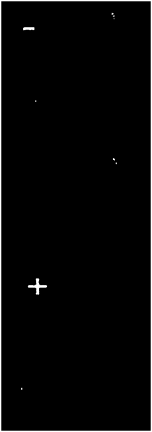
   e) Prove that $X=\{(x_1,x_2) \mid x_1 \le 5, x_2 \ge 3\}$ is a convex set.

   f) Write the dual of the following L.P.P:

   $$\text{Max } Z = CX$$
   $$\text{subject to } AX \le b$$
   $$X \ge 0.$$

   g) What are the assumptions made in the theory of games?

[Turn Over]



b) i) Find the value of the game and the optimal strategy for each player of the game whose pay-off matrix is given below: 7

B

		B_1	B_2	B_3
A	A_1	1	−1	−1
	A_2	−1	−1	3
	A_3	−1	2	−1

ii) Solve the following game problem: 3

B

		B_1	B_2
A	A_1	−2	5
	A_2	7	−6

c) i) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	a_i
O_1	3	7	2	1	11
O_2	9	4	7	3	20
O_3	10	2	8	3	35
b_j	10	5	21	30	

ii) Solve the following assignment problem.

	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

OPTION–B

MATH-G-DSE-T-02B

(Numerical Methods)

1. Answer any **ten** questions : 2×10=20

a) If $f(-2) = f(3) = 7$ and $f(0) = 1$, find $f(10)$.

b) What do you mean by the degree of precision of a quadrature formula?

c) Show that $\Delta\{\log f(x)\} = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$.

d) If $f(x)=e^{ax-b}$, prove that $f(0)$, $\Delta f(0)$, and $\Delta^2 f(0)$ are in G.P.

e) Find the relative error the computation of $y = x^3 + 3x^2 - x$, for $x = \sqrt{2}$, taking $\sqrt{2} = 1.414$.

f) State the fundamental theorem of the calculus of finite differences.

g) Find the minimum number of iterations required to attain an accuracy of 0.001 in an interval [1, 2] using bisection method.

h) Describe geometrically the convergence of the method of false position.

i) The Trapezoidal rule applied to $\int_1^3 f(x)\,dx$ gives the value 8 and the Simpson's one-third rule gives the value 4. Find $f(2)$.

j) Apply Runge-Kutta method of fourth order to find an approximate value of $y(0.2)$, given that $\frac{dy}{dx} = x + y$ and $y(0) = 1$.

k) Using Newton-Raphson method obtain the root of $x^3 - 8x - 4 = 0$ correct upto two decimal places (Take the initial approximation as $x_0 = 0$).

l) Prove that ∇ is a linear operator.

m) What is meant by the diagonally dominant for the system of linear equations?

n) State the basic principle of Newton-Raphson method.

o) Find the iterative formula for Newton-Raphson method to find the square root of $\sqrt{m}$.

2. Answer any **four** questions: 5×4=20

a) Apply Gauss-Seidel iteration method solve the system of equation:

$$8x - y + z = 18$$
$$x + y - 3z = -6$$
$$2x + 5y - 2z = 3$$

Continue iterations until two successive approximations are identical when rounded to three significant digits.

b) If $y = 3x^7 - 6x$, find the percentage error in y at $x = 1$ if error in x is 0.05.

c) Find $y(4.4)$, by Euler's Modified Method, taking $h=0.2$, from the differential equation $\frac{dy}{dx} = \frac{2-y^2}{5x}$, $y(4) = 1$.

d) Discuss the method of iteration for numerical solution of an algebraic and transcendental equation.

e) Find a real root of $x^3 - 8x - 4 = 0$ between and 4 by using Newton-Raphson method correct upto four decimal places.

f) Establish Newton's forward interpolation formula.

3. Answer any **two** questions: 10×2=20

a) i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$ for the function $y = f(x)$ given in the table.

x	1	2	3	4	5	6
y	2.7183	3.3210	4.0552	4.9530	6.0496	7.3891

ii) Let $f(x)=0$ has real root in an interval [a, b] where $f(x)=0$ can be rewritten as $x=g(x)$. Then prove that the function $y=g(x)$ has a fixed point $\bar{x} = g(\bar{x})$ in [a, b] if $|g(x')| \le c < 1$, for x in $[a, b]$.

b) i) By integrating Newton's forward interpolation formula, obtain the basic form of Simpson's $\frac{1}{3}$rd rule for numerical integration.

ii) Evaluate $\int_0^2 \frac{1}{x^3+x+1}dx$ by Simpson's $\frac{1}{3}$rd rule with $h = 0.25$.

c) i) Use Euler's method to approximate the solution of $\frac{dx}{dt} = tx^3 - x$ $(0 \leq t \leq 1)$, $x(0) = 1$ over the interval [0, 1] using four steps.

ii) Evaluate $y(1)$ from the differential equation $\frac{dy}{dx} = x^2 + y$ with $y(0)=1$, taking $h=0.5$ by the fourth oredr Runge-Kutta method and hence, compare it to original solution.

d) i) Establish Lagrange's polynomial interpolation formula.

ii) If $x_1, x_2, ..., x_n$ be the interpolating points and $l_i(x) (i = 0, 1, 2, ..., n)$ be the Lagrangian functions, then show that $\sum_{i=0}^{n} l_i(x) = 1$.
