U.G. 1st Semester Examination - 2024

PHYSICS

[MINOR]

Course Code : PHY-MI-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 30 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: 1×5=5

   a) State and explain the Green's theorem.

   b) Write down the conditions for a function $f(x)$ to be differentiable at every point. Check whether the function $f(x)=\frac{1}{(x-x_0)}$ is differentiable at every point in x.

   c) Show that $\vec{A}.\frac{d\vec{A}}{dt}=A\frac{dA}{dt}$.

d) Write down the degree and order of the differential equation

$$\frac{d^3y}{dx^3}=\sqrt[n]{1+\frac{d^2y}{dx^2}+\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2}.$$

e) Find the inverse of a matrix $A=\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$.

f) Define the Skew-Hermitian matrix with an example.

g) Find the value of $x\delta(x-3)$ at the point $x=3$ and $|x|>3$.

h) Show that $\vec{A}$ and $\vec{B}$ are parallel or anti-parallel if $\vec{A}\times\vec{B}=0$, where $|\vec{A}|$ and $|\vec{B}|$ are not zero.

GROUP-B

2. Answer any **three** questions: 5×3=15

a) If $\vec{r}$ is the position vector of a particle of mass m relative to O and $\vec{F}$ is the external force on the particle then $\vec{\tau}=\vec{r}\times\vec{F}$ is the torque of $\vec{F}$ at O. Show that $\vec{\tau}=\frac{d\vec{L}}{dt}$ where $\vec{L}=\vec{r}\times m\vec{v}$, the angular momentum of the particle.

b) Verify the Stokes's theorem of $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

c) Sove the equation $\frac{dy}{dx}=\frac{y^3+x^2y}{x^3}$.

d) A non singular matrix **A** has eigen values λ_i and eigenvector $\mathbf{x}^i$. Find the eigenvalues of the inverse matrix $\mathbf{A}^{-1}$.

e) Show that

$$\delta\left[(x-x_1)(x-x_2)\right]=\frac{\left[\delta(x-x_1)+\delta(x-x_2)\right]}{|x_1-x_2|}.$$

f) Show that $y\cos(x)\,dx+\sin(x)\,dy=0$ is an exact differential equation and find its general solution. 2+3

GROUP-C

3. Answer any **one** question: 10×1=10

a) i) Solve $\frac{d^2y}{dx^2}+9y=\cos(2x)$. 5

ii) Show that the matrix $U=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is a unitary matrix. $2\frac{1}{2}$

iii) Prove that the eigenvalues of a hermitian matrix **H** are real. $2\frac{1}{2}$

b) i) If a vector $\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is ***irrotational*** then find the constant ***a, b, c***. 5

ii) Show that $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$, where A and B are two non-commuting matrices. $2\frac{1}{2}$

iii) Write down Stoke's theorem and explain it. $2\frac{1}{2}$
