U.G. 1st Semester Examination - 2024

MATHEMATICS

[MINOR]

Course Code : MATH-MI-T-1

(Algebra and Analytical Geometry)

[NEP-2020]

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The notations and symbols have their usual meanings.

1. Answer any **five** questions: $2\times5=10$

a) Give the polar representation of the complex number : $z=-\frac{1}{4}+i\frac{\sqrt{3}}{4}$.

b) Does the equation $x^2-y^2=0$ represent a pair of straight lines? If so find the angle between them.

c) Compute AB where $A=\begin{pmatrix}4 & -2\\ -3 & 0\\ 3 & 5\end{pmatrix}$ and $B=\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$.

[Turn over]

d) How many points are there on the ellipse $\frac{3}{r}=1-\frac{1}{2}\cos\theta$ whose radius vector is 4? Find the points.

e) Find the centre of the conic $x^2-2xy+8y^2+x-y+5=0$.

f) Define a relation ρ on Z by $a\rho b$ if and only if $3a+4b=7n$, for some integer n. Show that ρ is reflexive and symmetric.

g) Find the rank of the matrix $\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 0 & 1 & -1\end{pmatrix}$.

h) Find the value of $\alpha^2+\beta^2+\gamma^2$ if α, β, γ are the roots of $x^3-6x^2+11x-6=0$.

2. Answer any **two** questions: 5×2=10

a) Find a solution (if exists) of the following system of linear equations:

$x-3y+4z=-4$

$3x-7y+7z=-8$

$-4x+6y-z=7$.

b) Show that $n^n>1.3.5...(2n-1)$.

c) Prove that the equation $ax^2+2hxy+by^2=0$, $(a,h,b)\neq(0,0,0)$ will be a pair of straight lines through the origin if and only if $h^2-ab\geq 0$.

d) A sphere touches the planes $2x+3y-6z+14=0$ and $2x+3y-6z+42=0$ and its centre lies on the line $2x+z=0$, $y=0$. Find the equation of the sphere.

3. Answer any **two** questions: 10×2=20

a) i) If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$ then prove that $\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$. 5

ii) If α,β,γ are the roots of the equation $x^3-7x^2+x-5=0$, find the equation whose roots are $\alpha+\beta, \beta+\gamma, \alpha+\gamma$. 5

b) i) Let a and b be nonzero real numbers. Define $f: R\rightarrow R$ by $f(x)=\frac{ax}{x+b}$. Prove that f is one-to-one. Are there solutions to the equations $f(x)=a$ and $f(x)=b$? 2+2

ii) What do you mean by a subgroup of a group? Do you find a group structure in Z? If so then check whether N and $H=\{2n: n\in Z\}$ are subgroups of Z or not. Here Z = set of all integers and N = set of natural numbers. 1+1+2+2

c) i) If the expression $ax^2+2hxy+by^2$ is transformed to $a_1x_1^{\,2}+2h_1x_1y_1+b_1y_1^{\,2}$ by transformation of rotation then prove the following $ab-h^2=a_1b_1-h_1^{\,2}$. 4

ii) Show by reducing to the canonical form, the equation $x^2-2xy+y^2+6x-14y+29=0$ actually represents a parabola. 6

d) i) Find the value of m, so that the lines $\frac{x-1}{2}=\frac{y-4}{1}=\frac{z-5}{2}$ and $\frac{x-2}{-1}=\frac{y-8}{m}=\frac{z-11}{4}$ may intersect. 5

ii) Find the image of x-axis on the plane $x+2y+3z=14$. 5
