U.G. 4th Semester Examination - 2020

MATHEMATICS

[PROGRAMME]

Course Code : MTMP-CC-T-4

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) If (G, o) be a group and $a, b \in G$ then prove that $(aob)^{-1} = b^{-1}oa^{-1}$.

b) If each in a group be its own inverse then prove the group is abelian.

c) Prove that union of two subgroups not necessarily a subgroup.

d) Find all elements of order 10 in a group $(\mathbb{Z}_{30}, +)$.

e) Find all cyclic subgroups of Klein's 4-Group.

f) Prove that all proper subgroups of order 8 is commutative.

g) Prove that symmetric group S_3 has a trivial center.

h) If (G, o) be a group and $a \in G$. Prove that aG = G, where aG = $\{aog : g \in G\}$.

i) Prove that a group of order 27 must have a subgroup of order 3.

j) If a be an element of a group and o(a) = 20. Find the order of the element a^6.

k) Let G be a group and $a \in G$. Prove that $\langle a \rangle$ is a normal subgroup of C(a).

l) If G = {e, a, b, c, d} be a cyclic group with identity element e find the order of the element b.

m) Prove that in a Boolean ring R, $a+a = 0$ for every $a \in R$.

n) Give an example of a ring which is not an integral domain.

o) Give an example of a finite ring R with unity and a subring S of R containing no unity.

2. Answer any **four** questions: 5×4=20

a) Let (G, o) be a group and $a, b \in G$ suppose $a^2 = e$ and $aoboa = b^7$, prove that $b^{48} = e$.

b) Let $G \neq \{e\}$ be a group of order p^n, p is a prime. Show that G contains an element of order p.

c) If H be a normal subgroup of a group G such that $o(H) = 3, [G:H] = 10$. If $a \in G$ and o(a)=3, prove that $a \in H$.

d) State and prove a necessary and sufficient condition that a subgroup is a normal subgroup of a group.

e) In a ring R if $x^2 = x, \forall x \in R$, then show that R is commutative. Also show that $(a+b)^2 = a^2 + 2ab + b^2$ for all $a, b \in R$.

f) Show that the set of all units in a ring R with unity forms a group with respect to multiplication.

3. Answer any **two** questions: 10×2=20

a) i) Let G be a group in which $(ab)^3 = a^3b^3$ for all $a, b \in G$. Prove that $H = \{x^3 : x \in G\}$ is a normal subgroup of G.

ii) State and prove the Lagrange's theorem.

b) i) Let (G,*) be a group and H be a non-empty finite sub-set of G. Then show that (H,*) is a subgroup of (G,*) if and only if $a \in H, b \in H \Rightarrow a * b \in H$.

ii) Prove that the subgroup of a cyclic group is cyclic

c) i) Show that a finite integral domain is a field.

ii) If R is a commutative ring of prime characteristic p. Prove that $(a+b)^p = a^p + b^p$ for all $a, b \in R$.

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