U.G. 4th Semester Examination - 2020

MATHEMATICS

[GENERIC ELECTIVE]

Course Code : MATH(H)-GE-T-02

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Define Wronskian.

b) Find the degree and order of the differential equation $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{2}{3}}=\frac{d^2y}{dx^2}$.

c) Find the value of β so that the following differential equation is an exact differential equation

$$(x^3y^2+\beta xe^y+logx)dx+\left(\frac{1}{2}x^4y-6ysiny^2+x^2e^y\right)dy=0.$$

d) Find the integrating factor of the differential equation $(x^2-1)\frac{dy}{dx}=-2xy+x, x>1$.

e) Solve: $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}$.

f) When is a differential equation of the form $Pdx+Qdy+Rdz=0$ said to be integrable?

g) If $y=e^{2x}$ is a solution of the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+ky=0$, find the value of k.

h) Find the solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi(\frac{y}{x})}{\phi'(\frac{x}{y})}$.

i) Show that the general solution of the differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$ tends to zero as $t\to\infty$.

j) Find the differential equation of family of curves $y^2=4a(x+a)$, where a is a parameter.

k) Find the equation of the curve satisfying the differential equation $(x^2+1)\frac{d^2y}{dx^2}=2x\frac{dy}{dx}$ and passes through the point (0,1) having slope of the tangent at $x=0$ is 6.

l) If $y_1=xe^x$ and $y_2=x^2$ are both solutions of the differential equation $\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=0$, show that y_1 and y_2 is a pair of fundamental solutions.

m) Find a partial differential equation by eliminating the arbitrary function ϕ from the relation $xyz = \phi(x + y + z)$.

n) Find the differential equation of the planes having equal x and y intercepts.

o) Show that the differential equation $z_{xx} + 2z_{xy} + cosec^2 y z_{yy} = 0$ elliptic.

2. Answer any **four** questions: 5×4=20

a) Solve by the method of variation of parameters

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = \frac{e^x}{1+e^{-x}}.$$

b) Solve:

$$\left(e^x y + e^z\right)dx + \left(e^y z + e^x\right)dy + \left(e^y - e^x y - e^y z\right)dz = 0.$$

c) Solve the partial differential equation $2z + p^2 + qy + 2y^2 = 0$ by Charpit's method.

d) Solve the following system of simultaneous equations:

$$\frac{dx}{dt} + 9\frac{dy}{dt} + 2x + 31y = e^t$$

$$3\frac{dx}{dt} + 7\frac{dy}{dt} + x + 24y = 3.$$

e) If $Mdx + Ndy = 0$ is an exact differential equation and M, N are homogeneous function of degree $n (n \neq -1)$. Then show that $Mx + Ny = c$ is the complete primitive of the given differential equation.

f) Solve the differential equation $(2x+1)(x+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = (2x+1)^2$, if $y = x$ and $y = \frac{1}{(x+1)}$ are two linearly independent solutions of the corresponding homogeneous differential equation.

g) Find the general solution of the following differential equation:

$$p^3 - p^2(x^2 + xy + y^2) + p(x^3y + x^2y^2 + xy^3) - x^3y^3 = 0.$$

3. Answer any **two** questions: 10×2=20

a) i) Solve:

$$(2x+1)^3\frac{d^3y}{dx^3} - 3(2x+1)^2\frac{d^2y}{dx^2} + 5y = \log(2x+1).$$

ii) Find general and singular solution of $y = px + \sin^{-1} p$.

b) i) Solve: $\frac{dy}{dx} + \frac{1-2x}{x^2}y = 1$.

ii) Solve: $\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}$.

c) i) Reduce the differential equation $axyp^2 + (x^2 - ay^2 - b)p - xy = 0$ to the Clairaut's form and hence solve it.

ii) Solve the differential equation: $(D^2 + 2)y = x^2 e^{3x} + e^x cos2x$.

d) i) Solve the partial differential equation by Lagrange's Method: $(3x + y - z)p + (x + y - z)q = 2(z - y)$.

ii) Find the canonical form of the differential equation $yz_{xx} + z_{yy} = 0, y > 0$.

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