U.G. 3rd Semester Examination - 2019

PHYSICS

[HONOURS]

Course Code : PHYS(H)CC-05-T

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Answer any **five** questions: 2×5=10

a) State Dirichlet's conditions for a Fourier series.

b) What do you mean by the orthogonality special functions?

c) Evaluate $\Gamma\left(-\frac{1}{2}\right)$.

d) Prove the following property of the Beta function $\beta(l, m) = \beta(m, l)$.

e) What are the singular points of a second order linear differential equations?

f) What do you mean by random error?

[Turn over]

g) Write down the Parseval's formula.

h) Write down the relation between the Beta and Gamma function.

GROUP–B

2. Answer any **two** questions: 5×2=10

a) i) When does a Laguerre function transform to a Laguerre Polynomials? 2

ii) Find the constant a_b of the Fourier series for the function f(x)=x in $0 \le x \le 2\pi$. 3

b) i) Evaluate $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}} dx$. 2

ii) Find the regular singular points of the differential equation

$$2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + \left(x^2 - 4\right)y = 0 \qquad 3$$

c) Prove the orthogonality condition of Legendre Polynomials

$$\int_{-1}^{+1} P_m(x) P_n(x)\, dx = 0, \quad m \ne n \qquad 5$$

d) Write down the Bessel's differential equation of n-th order and its solution $J_n(x)$. 5

GROUP–C

3. Answer any **two** questions: 10×2=20

a) i) Find the Fourier series for the function $f(x) = e^{ax}$ for $0 < x < \pi$, where a is constant. 5

ii) Find the integral $\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^9 \theta \, d\theta$ using $\beta(m, n)$ function in terms of $\Gamma(x)$ function. 5

b) Using Froberius method, obtain a series solution in powers of x for differential equation:

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0 \text{ about } x=0.$$

10

c) i) Find three dimensional Laplace's equation in cylindrical co-ordinates.

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ii) Prove that

$$\int_{-1}^{+1} P_n(x)\left(1-2xt+t^2\right)^{-\frac{1}{2}} dx = \frac{2t^n}{2n+1}, \text{ given,}$$

$$\int_{-1}^{+1} \left[P_n(x)\right]^2 dx = \frac{2}{2n+1} \quad \text{where n is a}$$

positive integer. 4

d) A tightly stretched string with fixed end points at x=0 and x=l is initially in a position given by

$$y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$$

If it is released from rest from position x (within $0 < x < l$), find the displacement $y(x, t)$. 10