U.G. 3rd Semester Examination - 2019

MATHEMATICS

[GENERIC ELECTIVE]

Course Code : MATH(H)GE-01-T

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols and Notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Give an example of 1st kind discontinuity.

b) Show that $\lim_{x\to 3}\frac{1}{x}=3$.

c) If f is differentiable at c and $f(c)\neq 0$, then show that $\frac{1}{f}$ is also differentiable at c.

d) Show that the function $f(x, y)=(xy)^{\frac{1}{3}}$ is continuous at (0, 0).

e) Find the maximum value of $f(x)=2-|x|$.

f) For what value of a the curve $y=1-ax^2$ and $y=x^2$ cut orthogonally?

g) Examine whether that Rolle's theorem is applicable or not on the function

$$f(x)=\begin{cases}1-x^2, & x\leq 0\\ \cos x, & x>0\end{cases} \text{ in } \left[-1, \frac{\pi}{2}\right].$$

[Turn over]

h) Show that $\lim_{n\to\infty}\frac{\{(n+1)(n+2)\ldots 2n\}^{\frac{1}{n}}}{n}=\frac{4}{e}$.

i) Whether L'Hospital rule is applicable on $\lim_{x\to 0}\frac{\sin x}{x}$ or not?

j) Sketch the curve $r=a(1+\cos\theta)$.

k) Find the nth derivative of $e^{ax}\sin bx$.

l) Find the 1st order derivative of $\left(\tan^{-1}x\right)^{\log x}$ with respect to $\log x$.

m) Show that the asymptotes of the curve $x^2y^2=a^2\left(x^2+y^2\right)$ form a square whose side is of length $2a$.

n) Show that the function $f(x)=x^3$ has neither maximum nor minimum at $x=0$.

o) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

2. Answer any **four** questions: 5×4=20

a) Find the radius of curvature at the origin of the curve

$$5x^3+7y^3+4x^2y+xy^2+2x^2+3xy+y^2+4x=0.$$
5

b) State and prove the Rolle's Theorem. 5

c) Find the asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$. 5

d) Applying MVT prove that $\frac{x}{x+1} < \log(1+x) < x$, for all $x > 0$. 5

e) Consider the function $f(x, y)$ defined by

$$f(x, y) = \begin{cases} xy\frac{x^2 - y^2}{x^2 + y^2}, & where \quad x^2 + y^2 \neq 0 \\ 0, & where \quad x^2 + y^2 = 0. \end{cases}$$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. 5

f) If $y = \frac{\log x}{x}$, then prove that

$$y_n = \frac{(-1)^n n!}{x^{n+1}}\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} - \dots - \frac{1}{n}\right].$$ 5

3. Answer any **two** questions: 10×2=20

a) i) If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

5

ii) If u be a homogeneous function of x and y of degree n, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

5

b) i) If $a_0, a_1, a_2, \ldots, a_n$ are real numbers such that $\frac{a_0}{n+1}+\frac{a_1}{n}+\ldots+\frac{a_{n-1}}{2}+a_n=0$, then show that there exist at least one number x between 0 and 1 such that

$$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_n=0. \quad 4$$

ii) If $u(x,y)=\phi(xy)+\sqrt{xy}\,\psi\left(\frac{y}{x}\right)$, $x\neq 0,\ y\neq 0$, where ϕ and ψ twice differentiable function, prove that $x^2\frac{\partial^2 u}{\partial x^2}-y^2\frac{\partial^2 u}{\partial y^2}=0$. 6

c) i) State and prove the Lagrange's Mean-Value theorem. 4

ii) Find the expansion of $\log_e(1+x)$ in a power series of x and indicate the range of validity of the expansion. 6

d) i) Determine a, b, c such that

$$\lim_{x\to 0}\frac{x(a+b\cos x)+c\sin x}{x^5}=\frac{1}{60}. \quad 5$$

ii) Find the maximum and minimum value of $a\cos x+b\cos 2x$, in $-\frac{\pi}{4}\leq x\leq\frac{\pi}{4}$ where $a, b>0$. 5
