U.G. 3rd Semester Examination - 2019

MATHEMATICS

[HONOURS]

Course Code : MATH(H)CC-06-T

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Let $(G, \circ)$ be a group and $a \in G$. Prove that $aG=G$ where $aG = \{a \circ g : g \in G\}$.

b) If each element in a group be its own inverse then prove that the group is abelian.

c) Find all subgroups of the group $(Z, +)$.

d) Show that every proper subgroup of a group of order 6 is cyclic.

e) Find all even premutations on the set $\{1,2,3,4\}$.

f) Find all cyclic subgroups of the symmetric group S_3.

g) Let $(G, \circ)$ be a group and $a \in G$. Prove that Z(G), the centre of the group is a subgroup of C(a), the centraliser of a.

[Turn over]

h) If the binary operation * be defined on I, the set of all integers by $a*b=a+b+1$, $a,b \in I$ find the identity element with respect to the operation *.

i) Find the premulative group which is isomorphic to the group $G = (\{1, i, -1, -i\}, \circ)$.

j) G is a cyclic group of order 10 and G′ a cyclic group of order 5. Show that there exists a homomorphism ϕ of G onto G′ with $\phi(\ker \phi) = 2$.

k) Prove that every subgroup of Z(G) is a normal subgroup of G.

l) Let G be a group and the mapping $\alpha : G \to G$ is defined by $\alpha(x) = x^{-1}$, $x \in G$. Then show that α is an automorphism if and only is G if abelian.

m) Find the number of inner automorphisms of the group S_3.

n) Let G be a non-abelian group of order p^3 where p is a prime. Then show that O(Z(G)=p.

o) Prove that the symmetric group S_3 has a trivial centre.

2. Answer any **four** questions: 5×4=20

a) Let $(G, \circ)$ be a finite cyclic group of order $n > 1$, generated by a. Then show that for a positive integer r, a^r is also a generator of the group if and only if r is less than n and prime to n.

b) If each non-identity element in a group G be of order 2 then prove that $O(G)=2^n$ for same natural number n.

c) Show that a cyclic group of finite order n has one and only one subgroup of order d for every positive divisor d of n.

d) Let G be a cyclic group of order 12 generated by a and H be the cyclic subgroup of G generated by a^4. Prove that H is normal in G. Write down all cosets of H in G. Verify that the quotient group G/H is a cyclic group of order 4. 2+1+2

e) State and prove Fundamental theorem of homomorphism. 1+4

f) Let G be a group. Then Inn(G) is a normal subgroup of Aut(G)– Justify.

3. Answer any **two** questions: 10×2=20

a) i) Let $(S, \circ)$ be a semigroup with a right identity element e. If for every two distinct elements $a, b \in S$ there exists a unique x in s such that $a \circ x = b$, then prove that $(S, \circ)$ is a group.

ii) Let $(S, \circ)$ be a semi group. If for $x, y \in S$, $x^2 o y = y = y o x^2$ then prove that $(S, \circ)$ is an abelian group.

b) i) Let G be the group of all n×n real non-singular matrices and H be the group of all n×n real orthogonal matrices. Prove that H is a subgroup of G but H is not a normal subgroup of G.

ii) Let M and N be normal subgroups of a group G that $M \cap N = \{e\}$. Show that mn=nm for all $m \in M$ and $n \in N$. 6+4

c) i) Let $G = \langle a \rangle$ be a finite cyclic group of order n. Prove that Aut(G) is a group of order $\phi(n)$ where $\phi(n)$ is the number of positive integers less than n and prime to n.

ii) Show that two finite cyclic groups of the same order are isomorphic. Further prove that two infinite cyclic groups are isomorphic. 4+6

d) i) Let $(G, *)$ be a group and $a, b \in G$. If $a^2=e$ and $a * b^2 * a = b^3$ then prove that $b^5=e$.

ii) In a group $(G, \circ)$, $a^{n+1}b^{n+1}=b^{n+1}a^{n+1}$ and $a^n b^n=b^n a^n$ hold for all $a, b \in G$ and for some integer n. Prove that the group is abelian. 4+6
