U.G. 3rd Semester Examination - 2019

MATHEMATICS

[PROGRAMME]

Course Code : Math(G)CC-03-T

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings.

1. Answer any **ten** questions: $2\times10=20$

   i) Find the infimum and supremum of the set $A=\left\{\frac{1}{p}+\frac{1}{q}: p, q \in N\right\}$.

   ii) Show that N, the set of Natural number is unbounded above.

   iii) Show that [a, b] is not an open set.

   iv) Define limit point of a set. Is there a limit point of a finite set?

   v) Prove that R is closed as well as open.

[Turn over]

vi) Define countable set. Give an example of an uncountable set.

vii) Define convergence of a sequence. Give an example of a bounded sequence which is not convergent.

viii) Define a Cauchy sequence. Give an example of a Cauchy sequence.

ix) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\ldots+\frac{n}{n+1}+\ldots$ does not converge.

x) Define Cauchy criteria for convergence of a series.

xi) Give an example of a geometric series. Is $\sum \frac{1}{n}$ is convergent?

xii) Define alternating series. Give an example.

xiii) Give an example of a set which has three limit points. Is the set Q is closed?

xiv) Evaluate the limit of the sequence $\left\{\sqrt{(n+1)}-\sqrt{n}\right\}$.

xv) Show that $\sum \frac{n^n}{n!}$ does not converge.

2. Answer any **four** questions: 5×4=20

i) State and prove Archimedian property of R.

ii) Prove that a convergence sequence is bounded.

iii) Prove that every subset of a countable set is countable.

iv) Prove that every absolutely convergent series is convergent.

v) Prove that the set of natural numbers has no limit point.

vi) Is $\lim x_n = l$, then prove that $\lim \frac{(x_1 + x_2 + \ldots + x_n)}{n} = l$.

3. Answer any **two** questions: 10×2=20

i) a) Show that (0, 1) in R is not countable.

b) Find the derived set of $S = \left\{\frac{1}{n};\ n \in N\right\}$.

5+5

ii) a) State and prove Cauchy's general principle of convergence.

b) Prove that the p-series is convergent if p>1. 5+5

iii) a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

b) Prove that every infinite set has a countable subset. 5+5

iv) a) A sequence of function $\{f_n\}$ is defined on $[0, a]$, $0<a<1$, by $f_n(x)=x^n$, $x \in [0, a]$. Show that the sequence $\{f_n\}$ converges uniformly on $[0, a]$.

b) Prove that the series $\sum \frac{x}{n+n^2x^2}$ is uniformly convergent for all real x.

5+5
