U.G. 2nd Semester Examination - 2020

PHYSICS

[PROGRAMME]

Course Code : PHYG-CC-T-02

Mathematical Physics-II

Set–III

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

a) What is periodic function?

b) State Dirichlet's condition for the expansion of a function in Fourier series.

c) What do you mean by singular point?

d) When we are using the Frobenius method to get the solution of a differential equation?

e) State the orthogonality condition of the roots of a Bessel function.

f) Determine the value of Legendre Polynomial $P_n(1)$.

g) The length of a simple pendulum is increased by 2%. By what percentage will the frequency of oscillation increase or decrease?

h) Show that $\beta(m,n) = \beta(n,m)$.

2. Answer any **two** questions: 5×2=10

a) Write the integral form of $\Gamma(n)$. Show that

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$ 1+4

b) Expand the Fourier series of periodic function $f(x) = |x|$ in the interval $(-l, l)$. 5

c) Prove that $\int_{-1}^{+1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2-1}$. Write down the expression of Hermite differential equation. 4+1

d) Solve the differential equation $\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}$, where c is a constant.

Show that the recurrence relation of Legendre polynomial is

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x).$$

3+2

3. Answer any **two** questions: 10×2=20

a) What do you mean by orthogonal functions? Show that the orthogonal condition between two Legendre polynomials can be expressed as

$$\int_{-1}^{1} P_m(x) P_n(x) = \begin{cases} 0 \text{ for } m \neq n, \\ \frac{2}{2m+1} \text{ for } m = n \end{cases}.$$

Show that $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) = \frac{2\pi}{\sqrt{3}}$. 2+5+3

b) Write down the Bessel's differential equation of order n. Is there exists any singular point in this equation? What kind of singularity is this? Write down the general form of $J_n(x)$ (a solution of Bessel's equation). Show that the coefficient of t^n in the series expansion of $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ represent $J_n(x)$. 2+1+1+2+4

c) What is odd function? Find the Fourier series expansion of periodic function of period 2π defined as

$$f(x) = x \text{ if } -\frac{\pi}{2} < x < \frac{\pi}{2}.$$
$$= (\pi - x) \text{ if } \frac{\pi}{2} < x < \frac{3\pi}{2}$$

Find the Fourier series expansion of the periodic function $f(x) = x^2$ in the interval $-\pi \le x \le \pi$. Hence find the sum of the series

$\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$. (1+4)+(4+1)

d) What is separation of variable method? Set up the partial differential equation in two-dimensional Cartesian coordinate for Laplace equation $\nabla^2 u(x, y) = 0$. Solve the equation to get the general solution. Reduce the form of this solution using the boundary conditions $u(0, y) = 0$, $u(a, y) = 0$, $u(x, 0) = A(1-x/a)$ and $u(x, \infty) = 0$ in the region $0 \le x \le a$, $0 \le y \le \infty$. 1+1+5+3

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