U.G. 2nd Semester Examination - 2020

MATHEMATICS

[HONOURS]

Course Code : MTMH-CC-T-03

Full Marks : 60 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

The symbols and notations have their usual meanings.

1. Answer any **ten** questions: 2×10=20

a) Prove that N the set of natural numbers is an infinite set.

b) Show that $Cl(A\cup B)=Cl\,A\cup Cl\,B$, where $Cl\,A$ denotes the closure of A.

c) Give an example to show that $(A\cup B)^{o}$ may not be equal to $A^{o}\cup B^{o}$, where A^{o} denotes the interior of A.

d) State Peano's axioms on natural numbers.

e) Determine the cluster point(s) of the set: $\left\{-\frac{1}{2},1\frac{1}{2},-\frac{2}{3},1\frac{2}{3},-\frac{3}{4},1\frac{3}{4},...\right\}$.

[Turn Over]

f) If $\sum_{n=1}^{\infty}a_n$ is convergent series of positive terms, show that $\sum_{n=1}^{\infty}a_n^2$ is convergent.

g) Define countable set. Give an example of a countable set in R with uncountable many limit points.

h) Let $S=\left\{1-\frac{(-1)^n}{n}:n\in N\right\}$. Find inf S and sup S.

i) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^p}, p>1$.

j) If $\{a_n\}$ is a convergent sequence of real numbers, then test the convergence of $\{a_n+(-1)^n\}$.

k) Establish Archimedean property of real numbers.

l) Find the limit points of $\{2\pm n\}$, where n is a natural number.

m) Let $A=\{-1,2\}\cup\left\{\pm1+\frac{1}{n};n=1,2,3...\right\}$. Find greatest lower bound and least upper bound of the set A.

n) Let $A=\left\{sinx:x\in\left[0,\frac{\pi}{2}\right]\right\}$. Is the set A closed? Support your answer.

o) Give an example of an open set which is not an interval.

2. Answer any **four** questions: 5×4=20

a) Using well-ordering principle deduce the Principle of Mathematical Induction.

b) Show that no positive integer m other than a square number has a square root within the system $\boldsymbol{Q}$ of rational numbers.

c) Show that every subset of a countable set is countable. What will be the case for superset?

d) State the least upper bound axiom for real numbers. Deduce that a set of real numbers which is bounded bellow has the greatest lower bound.

e) Show that every convergent sequence is bounded. Is the converse true? Justify.

f) Show that a sequence $\{x_n\}$ of real numbers converges if and only if it is a Cauchy sequence.

3. Answer any **two** questions. 10×2=20

a) i) If x and y are two real numbers such that $x < y$, then show that there exists a rational number q such that $x < q < y$.

ii) If k is an approximation to $\sqrt{2}$, then prove that $\frac{4+3k}{3+2k}$ is a better approximation.

5+5

b) i) The function f is odd and g is a function whose domain is same as the range of f and the composite function $g_o f$ is odd. Prove that g is odd.

ii) Show that if a sequence of real numbers is monotone increasing and bounded above, then it converges to its exact upper bound. 5+5

c) i) Show that if a series is absolutely convergent, then the series formed by its positive terms alone is convergent, and the series formed by its negative terms alone is convergent.

ii) State and prove Cauchy's convergence criterion for infinite series of real numbers. 5+5

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