U.G. 1st Semester Examination - 2019

PHYSICS

[HONOURS]

Course Code : PHS(H)-CC-01-T

(Mathematical Physics-I)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

1. Answer any **five** questions: 2×5=10

   i) Plot the graph of $y = x^2$.

   ii) Define periodic function with example.

   iii) Show that limit $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

   iv) Obtain Taylor series expansion of $f(x) = e^x$ about $x = 0$.

   v) Define a Liamellar vector field.

   vi) Show that the orthonormal vector triad $(\vec{i}, \vec{j}, \vec{k})$ is self-reciprocal.

   vii) Show that the area bounded by a simple closed curve C is a plane and is given by $A = \frac{1}{2}\oint_C (xdy - ydx)$.

   viii) What are curvilinear coordinates? When are they called orthogonal?

[Turn Over]

Answer any **two** questions: $5\times2=10$

2. i) Show that $f(x) = \log\left(x+\sqrt{1+x^2}\right)$ is an odd function of x.

   ii) Evaluate: $\lim_{x \to 0} \frac{1-\cos^3 x}{\tan^2 x}$. 2+3

3. i) The function $f(x) = \frac{\ln(1+ax)-\ln(1-bx)}{x}$ is not defined as $x = 0$. Find the values of f(0), so that f(x) is continuous at x = 0.

   ii) If u = f(x+ct) + g(x–ct), then prove that it satisfies the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

   2+3

4. i) Define direction cosine of a vector. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two vectors, show that the angle θ between them is $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.

   ii) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}^*, \vec{b}^*, \vec{c}^*$ be the set of reciprocal vectors, then show that $\vec{a}^*.\left(\vec{b}^*\times\vec{c}^*\right) = \frac{1}{\vec{a}.\left(\vec{b}\times\vec{c}\right)}$.

   (1+2)+2

5. i) Find the acceleration of a particle moving along a space curve with velocity $\vec{v}$.

   ii) a) Prove that $\nabla^2\phi(r) = \frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi(r)}{dr}$.

b) Find $\phi(r)$ such that $\nabla^2\phi(r)=0$.

2+(2+1)

Answer any **two** questions: 10×2=20

6. i) State and prove Euler's theorem.

ii) Find the solutions of the differential equation $y''(x) - 3y'(x) + 2y(x) = 0$. Show that these solutions are linearly independent. Find the solution of $y(x)$ with the properties that $y(0) = 0$, $y'(0) = 1$.

iii) With the help of Lagrange's method of undetermined multiplier, show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

(1+2)+(1+1+1)+4

7. i) Prove that:

$$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right)=\left(\vec{B}.\vec{\nabla}\right)\vec{A}-\left(\vec{A}.\vec{\nabla}\right)\vec{B}+\vec{A}\left(\vec{B}.\vec{\nabla}\right)-\vec{B}\left(\vec{\nabla}.\vec{A}\right).$$

ii) If $\vec{\nabla}.\vec{E}=0$, $\vec{\nabla}.\vec{H}=0$, $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{H}}{\partial t}$,

$\vec{\nabla}\times\vec{H}=\frac{\partial\vec{E}}{\partial t}$, show that $\vec{E}$ and $\vec{H}$ satisfy the equation $\nabla^2 u=\frac{\partial^2 u}{\partial t^2}$.

iii) If $\vec{\nabla}\times\vec{A}=0$, then show that

a) $\vec{\nabla}.\left(\vec{A}\times\vec{r}\right)=0$.

b) Prove that: $\vec{\nabla}\times\left(\vec{A}\times\vec{r}\right)=2\vec{A}$.

c) Prove that $\left(\vec{A}\times\vec{\nabla}\right)\times\vec{r}=-2\vec{A}$.

3+2+(1+2+2)

8. i) State and prove Green's theorem in the plane.

ii) Find the work done in moving a particle in the force field $\vec{F}=3x^2\hat{i}+\left(2xz-y\right)\hat{j}+z\hat{k}$ along the straight line from (0, 0, 0) to (2, 1, 3).

iii) By using divergence theorem determine the value of the integral $\iint_S \left(xdydz+ydxdz+zdxdy\right)$ where S is the surface of the region bounded by the cylinder $x^2+y^2=9$ and the planes $z=0$ and $z=3$.

(1+3)+3+3

9. i) Calculate the scale factors h_ρ, h_θ, h_ϕ in spherical polar coordinates.

ii) Represent the vector $\vec{A}=2y\hat{i}-z\hat{j}+3x\hat{k}$ in cylindrical co-ordinates (ρ, ϕ, z) and find A_ρ, A_ϕ, and A_z.

iii) Show that the differential operators in terms of orthogonal curvilinear co-ordinates is

$$\vec{\nabla}=\frac{1}{h_1}\frac{\partial}{\partial u_1}\hat{e}_1+\frac{1}{h_2}\frac{\partial}{\partial u_2}\hat{e}_2+\frac{1}{h_3}\frac{\partial}{\partial u_3}\hat{e}_3.$$

2+(3+1)+4