U.G. 1st Semester Examination - 2023

MATHEMATICS

[Skill Enhancement Course (SEC)]

Course Code : MATH-SEC-T-01

(Logic & Boolean Algebra)

[NEP-2020]

Full Marks : 35 Time : $1\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Sysmbols have their usual meanings.

1. Answer any **five** questions : $1\times5=5$

a) Is the statement "How beautiful is Rose?" a proposition? Justify your answer.

b) Write the truth table for $p \leftrightarrow q$.

c) Define the universal quantifier with an example.

d) What is the difference between the converse and contrapositive of a proposition?

e) Express the negation of the statement "If the teacher is absent, then some students do not keep quiet" using quantifiers.

f) Define a lattice with an example.

[Turn over]

g) If $\{L, \leq\}$ is a lattice and $a \in L$, prove that $a \vee a = a$ and $a \wedge a = a$.

h) If $(B, +, ., ', 0, 1)$ is a Boolean algebra, prove that $a + 1 = 1$ and $a.0 = 0$.

2. Answer any **two** questions : 5×2=10

a) What is a tautology? Determine whether the compound proposition $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology. 1+4

b) Without using truth table, prove that $(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$. 5

c) Simplify the following Boolean expression $(x.y + z')(y' + z.x') + x'.y'.z'$ and then draw a switching circuit for it. 5

d) Simplify the Boolean function $f(a,b,c,d) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$ using Karnaugh map method. 5

3. Answer any **two** questions : 10×2=20

a) i) Express the statement "if a number is divisible by 3 and by 5, then it is divisible by 15" as a compound proposition and construct its truth table. 5

ii) Prove the following equivalence by proving the equivalence of the dual: $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$ 5

b) i) If $P(S)$ is the power set of a set S and $\cup$ and $\cap$ are taken as the join and meet, prove that $\{P(S), \subseteq\}$ is a lattice. 5

ii) What is a distributive lattice? In a distributive lattice $\{L, \vee, \wedge\}$, prove that if an element $a \in L$ has a complement, then it is unique. 1+4

c) i) For any two elements a and b in a Boolean algebra, show that $(a+b)' = a'.b'$. 5

ii) Convert the Boolean expression $(x.y' + y.z' + z.x')'$ in Disjunctive normal form involving three variables x, y and z. 5

d) i) Define a minterm with an example. 2

ii) Minimise the Boolean expression $f(a,b,c,d) = \Sigma(0, 1, 3, 8, 9, 13, 14, 15, 16, 17, 19, 24, 25, 27, 31)$ using Quine McCluskey method. 8