U.G. 1st Semester Examination - 2023

PHYSICS

[MAJOR]

Course Code : PHY-M-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any five questions : 2×5=10

a) Evaluate $\lim_{x\to 0}\frac{x-|x|}{x}$

b) State the order and degree of the differential equation

$$\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^3=3\cos(x)$$

c) Check whether the three vectors $\hat{\imath}, \hat{\imath}+\hat{\jmath}, \hat{\imath}+\hat{\jmath}+\hat{k}$ are linearly independent.

d) Check whether $dw=2xy\,dx+x^2dy$ is an exact differential.

e) Find $\vec{\nabla}(\vec{\nabla}.\vec{A})$ where $\vec{A}=\frac{\vec{r}}{r}$

[Turn over]

f) Find the eigen values of the matrix

$$A=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

g) Prove that $\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx = f(a)$

h) Using Gauss' divergence theorem, show that

$$\iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)dV = \oint\!\!\oint_S \left(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi\right).d\vec{S}$$

where $\phi(x, y, z)$ and $\psi(x, y, z)$ are two scalar functions and and the surface integral is over the surface S enclosing the volume V.

GROUP-B

2. Answer any **two** questions : 5×2=10

a) Sketch the function $e^{x}, e^{-x}, e^{-|x|}$ for $-1 \le x \le 1$. Explain whether the function $e^{-|x|}$ is differentiable at $x = 0$

b) Solve the equation

$y'' + 6y' + 8y = 0$

Subject to the condition $y = 1, y' = 0$ at $x = 0$

where $y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2y}{dx^2}$.

c) Considering two position vectors in three dimensions, show that their scalar product remains invariant under the rotation of co-ordinate system about z-axis.

d) Consider the vector field $\vec{F} = -5\hat{r} + \sin\phi\sin\theta\hat{\theta}$, Calculate curl $\vec{F}$.

GROUP-C

Answer any **two** questions : 10×2=20

3. a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

b) Suppose that the temperature T at any point (x, y, z) is given by

$T(x, y, z) = x^2 - y^2 + yz + 373.$

In which direction is the temperature increasing most rapidly at (–1, 2, 3)? What is the maximum rate of change of temperature at that point?

c) If S is any closed surface enclosing a volume V and $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$, show that

$\oint\!\!\oint_S \vec{A}.\vec{ds} = (a+b+c)V$. 3+(1+3)+3

4. a) Prove that $\vec{\nabla}.(\phi\vec{V}) = (\vec{V}\phi).\vec{V} + \phi(\vec{\nabla}\times\vec{V})$ for a scalar field $\phi(x,y,z)$ and a vector field $\vec{V}(x,y,z)$.

Now take $\vec{V}$ to be a non-zero constant vector field $\vec{C}$ and use Stoke's theorem to prove that,

$$\oint_C \phi \vec{dr} = \iint_S \vec{ds}.\vec{\nabla}\phi$$

where the closed curve C is the boundary of the surface S.

b) Find the values of λ, μ, ν so that the vector $\vec{F} = (x+\lambda y+4z)\hat{i} + (2x-3y+\mu z)\hat{j} + (\nu x - y + 2z)\hat{k}$ is conservative. Find also the scalar function $\phi(x,y,z)$ such that $\vec{F} = \vec{\nabla}\phi$. (3+3)+4

5. a) Solve $\sin x \frac{dy}{dx} + y\cos x = 2\sin^2 x \cos x$

b) Find the Jacobian of the transformation

$x=5v+5w^2$, $y=3w+3u^2$, $z=2u+2v^2$ 5+5

6. a) Show that the infinitesimal volume element in Spherical Polar Coordinate system (r,θ,ϕ) is $r^2 \sin\theta\, dr\, d\theta\, d\phi$.

b) Verify the divergence theorem for $\vec{A} = 4xz\hat{\imath} + y^2\hat{\jmath} + yz\hat{k}$ and a cube bounded by the planes

$x = 0, x = 1, y = 0, y = 1, z = 0$ *and* $z = 1$.

c) Prove that $\oint u\vec{\nabla}v.d\vec{r} = -\oint v\vec{\nabla}u.d\vec{r}$. 4+4+2
