U.G. 1st Semester Examination - 2023

PHYSICS

[MINOR]

Course Code : PHY-MI-T-01

(Mathematical Physics-I)

[NEP-2020]

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions : $2\times5=10$

a) The function $f(x)=|x-2|$ is continuous at x=2. Check whether the function is differentiable at x=2 or not.

b) Plot the function $y=2\sin\left(x-\frac{r}{2}\right)+3$, Find the time period of the function.

c) Discuss the exitance of unique solution of the ordinary differential equation $y'=xy-\sin y$, $y(0)=2$

d) Find the projection of $\vec{A}=\hat{\imath}-2\hat{\jmath}+\hat{k}$ on the vector $\vec{B}=4\hat{\imath}-4\hat{\jmath}+7\hat{k}$

e) Evaluate $Div\,(2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k})$

f) What is independent random variable? Give example.

g) Find the eigen values of the matrix $A=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$.

h) Give example of a symmetric and a skew-symmetric matrix.

GROUP-B

2. Answer any **two** questions : 5×2=10

a) State Greens theorem. Prove Greens theorem using Gause divergence theorem.

b) If $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate the flux of $\vec{A}$ across the surface of a unit cube with two opposite corners at (0,0,0) and (1, 1, 1) respectively.

c) Investigate whether the equation $x^2ydx-(x^3-y^3)dy=0$ is exact or not. Solve the equation.

d) Calculate the determinant, trace, eigenvalue and eigen vector of these Pauli matrices

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

GROUP-C

Answer any **two** questions : 10×2=20

3. a) Using Green's theorem evaluate $\int_C \left(x^2ydx - x^2dy\right)$ where C is boundary described counter-clock wise of the triangle with vertices (0, 0), (1,0) (1, 1).

b) If $\vec{F} = \left(2x^3 - 3z^2\right)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ find $\iiint \vec{\nabla}.\vec{F}dV$, where V is the region bounded by the co-ordinate planes and the plane $2x+2y+z=4$. 5+5

4. a) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are two vectors. Show that the scalar product $\vec{A}.\vec{B}$ remains invarient under rotation of angle θ about Y axis.

b) Let $y_1=t$ and $y_2=te^{4t}$ are two solutions of a second order differential equation. Show that these two solutions are independent. 6+4

5. a) Plot the curve y= cosh X

b) Show that $df=y(xy+e^x)dx-e^xdy$ is not exact differential but $\frac{df}{y^2}$ is exact.

c) Find the area of the ellipse $x=a\cos\varphi$, $y=b\sin\varphi$.

d) What is Dirac delta function? Write two properties of it.

e) Show that $\delta\left(\frac{x}{a}\right) = |a|\delta(x)$ 2+2+2+2+2

6. a) State the order and degree of the differential equation

$$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 + x^3y = 0$$

b) Solve the equation $y'' + 6y' + 8y = 0$

Subject to the condition $y = 1$, $y' = 0$ at $x = 0$

where $y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2y}{dx^2}$

c) Prove $\vec{\nabla}\times(\phi\vec{\nabla}) = (\vec{\nabla}\phi).\vec{V} + \phi(\vec{\nabla}.\vec{V})$ for a scalar field $\phi(x,y,z)$ and a vector field $\vec{V}(x,y,z)$.

Now take $\vec{V}$ to be a non-zero constant vector field $\vec{C}$ and use Stoke's theorem to prove that,

$$\int_C \phi d\vec{V} = \iint_S d\vec{s}.\vec{\nabla}\phi$$

where the closed curve C is the boundary of the surface S. 1+4+5