# U.G. 1st Semester Examination - 2023

## MATHEMATICS

## [MINOR]

### Course Code : MATH-MI-T-1

### (Algebra and Analytical Geometry)

### [NEP-2020]

Full Marks : 40 Time : 2 Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*The notations and symbols have their usual meanings.*

1. Answer any **five** questions : 2×5=10

 a) Find the modulus and principal value of the amplitude of $(\cos 50^\circ + i\sin 50^\circ)^6$.

 b) Find the remainder when $4x^5+3x^3+6x^2+5$ is divided by 2x+1.

 c) Express $A = \begin{pmatrix} 4 & 5 & 1 \\ 3 & 7 & 2 \\ 1 & 6 & 8 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

d) Find the order of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 6 & 5 & 4 & 2 \end{pmatrix}$$

e) Determine the nature of the conic

$x^2-2xy+2y^2-4x-6y+3=0$

f) Transform the equation $y^2-2y=x$ with respect to parallel axes through (–1, 1).

g) Find the points on the conic $\frac{5}{r}=1+2\cos\theta$, whose radius vector is 5.

h) Does the equation $x^2+3xy+2y^2=0$ represent a pair of straight lines? If so, find the angle between them.

2. Answer any **two** questions : 5×2=10

a) Prove that the sum of 99th powers of all the roots of $x^7-1=0$ is zero.

b) Solve the following system of equations by matrix method.

$$x+z=0$$
$$3x+4y+5z=0$$
$$2x+3y+4z=1$$



c) If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, then prove that $pq=-1$.

d) Show that the condition that the line $\frac{l}{r}=a\cos\theta+b\sin\theta$ may touch the conic $\frac{l}{r}=1+e\cos\theta$ is $(la-e)^2+l^2b^2=1$.

3. Answer any **two** questions : 10×2=20

a) i) Find the general and principal value of $(1+i)^{1-i}$.

ii) Apply Descarte's rule of sign to examine the nature of roots of the equation.

$x^4+2x^2+3x-1=0$. 6+4

b) i) Determine the rank of the matrix

$$A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{pmatrix}$$

ii) On the set of integers $\mathbb{Z}$, the binary operation * is defind by $a*b=a+b-2$ for all $a,b\in\mathbb{Z}$. Show that $(\mathbb{Z},*)$ is a group.

5+5

c) i) Prove that the pair of the straight lines joining the origin to the points of intersection of the curves

$$ax^2+2hxy+by^2+2gx=0 \text{ and}$$

$$a'x^2+2h'xy+b'y^2+2g'x=0$$

are perpendicular if $g'(a+b)=g(a'+b')$.

ii) Reduce the following equation in canonical form and determine the nature of the conic

$$x^2-2xy+2y^2-4x-6y+3=0.$$

5+5

d) i) Find the equation of the cone whose vertex is at the point (6, 0, 0) and whose generators touch the surface $x^2+y^2+z^2=25$.

ii) If the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents two straight lines, then prove that the product of the lengths of the perpendiculars from the origin on these straight line is

$$\frac{c}{\sqrt{(a-b)^2+4h^2}}$$

5+5